METU - NCC

	Precalculus Midterm	
Code : *Math 100* Acad.Year : *2014-2015* Semester : *Fall* Date : *18.11.2012* Time : *17:40* Duration : *100 minutes*	Last Name: Name : Department: Signature:	Student No.: Section:
	10 QUESTIONS ON 4 PAGES TOTAL 100 POINTS	

1 (10)	2 (8)	3 (12)	4 (10)	5 (10)	6 (10)	7 (10)	8 (8)	9 (10)	10 (12)			

1. *(10 pts)* Find all x satisfying $|3x-12| - x \le |2x|$.

when $x<0$; $-3x+12-x \le -2x \Rightarrow 12 \le 2x \Rightarrow 6 \le x \Rightarrow$ No solution

when $0<x<4$; $-3x+12-x \le 2x \Rightarrow 12 \le 6x \Rightarrow 2 \le x \Rightarrow$ solution ; $[2,4)$

$\boxed{\text{Solution: } [2,\infty)}$

when $4<x$; $3x-12-x \le 2x \Rightarrow -12 \le 0$

$\Rightarrow$ solution; $(4,\infty)$

when $x=0$; $|3\cdot 0-12| - 0 \le |2\cdot 0| \Rightarrow 12 \le 0$ Not true.

when $x=4$; $|3\cdot 4-12| - 4 \le |2\cdot 4| \Rightarrow -4 \le 8$ True.

2. *(8 pts)* If the difference between the average of squares of two numbers and the square of average of these two numbers is 9 then find their difference.

$$\frac{a^2+b^2}{2} - \left(\frac{a+b}{2}\right)^2 = 9 \Rightarrow \frac{a^2+b^2}{2} - \frac{a^2+b^2+2ab}{4} = 9$$

$\boxed{-6 \text{ or } +6}$

$$\Rightarrow \frac{2a^2+2b^2-a^2-b^2-2ab}{4} = 9$$

$$\Rightarrow a^2+b^2-2ab = 36 \Rightarrow (a-b)^2 = 36 \Rightarrow a-b = \mp 6$$

3. *(12 pts)* Find all solutions of x satisfying

$$(x^2-2x)^2 - 4(x^2-2x) + 3 = 0$$

say $x^2-2x = a \Rightarrow a^2-4a+3 = 0 \Rightarrow (a-3)(a-1) = 0$

$a=1 \Rightarrow x^2-2x = 1 \Rightarrow x^2-2x-1 = 0 \Rightarrow (x-1)^2 = 0 \Rightarrow x = 1.$

$a=3 \Rightarrow x^2-2x = 3 \Rightarrow x^2-2x-3 = 0 \Rightarrow (x-3)(x+1) = 0 \Rightarrow x = -1$ or 3

$\boxed{\text{solutions: } -1, 1, 3}$

4. *(10 pts)* Which one(s) of the following data may belong to a linear function and which one(s) of them can not belong to a linear function? Explain. For those which may belong to a linear function find their formula.

x	1	2	4	5
$f(x)$	-3	0	6	9
$g(x)$	1	4	9	16
$h(x)$	-2	1	7	16

Check slopes for each interval

Interval	f(x)	g(x)	h(x)
1-2	$\frac{0-(-3)}{2-1}=3$	$\frac{4-1}{2-1}=3$	$\frac{1-(-2)}{2-1}=3$
2-4	$\frac{6-0}{4-2}=3$	$\frac{9-4}{4-2}=\frac{5}{2}$	$\frac{7-1}{4-2}=3$
4-5	$\frac{9-6}{5-4}=3$	$\frac{16-9}{5-4}=7$	$\frac{16-7}{5-4}=9$

f(x) has the same slope for all intervals, so f may belong to a linear function but others cannot.

$f(x) = 3x+n$ and $f(1) = -3 \Rightarrow -3 = 3\cdot 1+n \Rightarrow n=-6 \Rightarrow f(x) = 3x-6$

5. *(10 pts)* Rewrite the quadratic function $P(x) = (2-x)(x-8)$ in the vertex form $P(x) = a(x-h)^2+k$. Does $P(x)$ have a maximum or a minimum value at its vertex? Explain.

$P(x) = (2-x)(x-8) = -x^2+10x-16$

$\Rightarrow P(x) = -(x^2-10x)-16$

$= -((x-5)^2-25)-16$

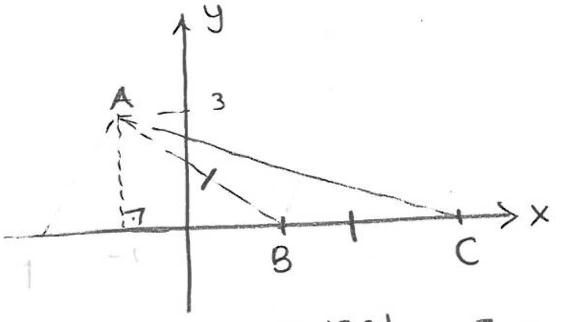
$P(x) = -(x-5)^2+9$

Vertex form: $P(x) = -(x-5)^2+9$

at its vertex, $P(x)$ *has* .max. *value*

Since $a=-1$; it looks down and hence at its vertex $P(x)$ has max value.

6. *(10 pts)* The vertices of a triangle ABC are $A = (-1,3)$, $B = (b,0)$, $C = (c,0)$ such that $0 < b < c$. If $d(A,B) = d(B,C)$ (i.e. if ABC is an isoceles triangle) and if $area(ABC) = 7.5\ unit^2$, find the x-coordinates of B and C.



$b = 3$

$c = 8$

$\text{Area} = \frac{3\cdot|BC|}{2} = 7.5 \Rightarrow |BC| = 5 \Rightarrow |AB| = 5$

$d^2(A,B) = (b-(-1))^2+(0-3)^2 = 5^2 \Rightarrow b+1 = \mp 4 \Rightarrow \cancel{b=-5}$ or $b=3$

$b>0$

$d^2(B,C) = (c-b)^2+(0-0)^2 = 5^2 \Rightarrow c-b = \mp 5 \Rightarrow c-3=-5$ or $c-3=5$

$\cancel{c=-2}$ $\quad c=8$

$c>0$

7. *(10 pts)* What is the equation of the line which passes through the y-intercept of the curve $y = x^3 + x^2 - x + 1$ and is perpendicular to the line $y = -\frac{x}{2} - 4$.

Slope of a line perpendicular to $y=-\frac{x}{2}-4$ is $\frac{-1}{-\frac{1}{2}} = 2$.

y-intercept of $y=x^3+x^2-x+1$ is $y_{int}=1$, when $x=0$.

So, the line eqn: $\frac{y-1}{x-0}=2$ or $y=2x+1$

$\boxed{y=2x+1}$

8. *(8 pts)* Complete the table below:

x	1	2	3	4
$f(x)$	4	1	4	3
$g(x)$	2	4	1	5
$(f \circ f)(x)$	3	4	3	4
$(g \circ f)(x)$	5	2	5	1

9. *(10 pts)* Find the domain of the function

$$f(x) = \frac{7}{\sqrt{|x+1| + x}}$$

$|x+1|+x>0$

when $x<-1$; $-x-1+x>0 \Rightarrow -1>0$ wrong

when $x>-1$; $x+1+x>0 \Rightarrow x>-\frac{1}{2}$

when $x=-1$; $|-1+1|-1>0$ wrong

Domain $f(x) = \boxed{(-\frac{1}{2}, \infty)}$

10. *(12 pts)* Graph of the function $g(x) = 2f(-x) - 2$ is given below. Using basic trasformations sketch the graph of $h(x) = f(x+1)$ in the blank grid. Show individual steps of the transformation of the graph of $f(x)$ to the graph of $g(x)$.